RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

SECOND YEAR [BATCH 2016-19]
B.A./B.Sc. FOURTH SEMESTER (January – June) 2018
Mid-Semester Examination, March 2018

Date : 14/03/2018 **MATHEMATICS (Honours)**
Time : 2 pm – 4 pm Paper : IV Full Marks : 50

[Use a separate Answer Book for each group]

Group – A [25 marks]

Answer any three questions from Question Nos. 1 to 5 : [3×5]

1. Let $X = P(\mathbb{N})$, the power set of $\mathbb{N}$. Define $d: X \times X \to \mathbb{R}$ by

$$d(A,B) = \begin{cases} 0, & A \Delta B = \phi \\ \frac{1}{m}, & m \text{ is the least element of } A \Delta B \end{cases}$$

Prove that (X,d) is a metric space.

2. Define a G_δ-set in a metric space. Is the set $A = (0,1] \cup \{2\}$ is G_δ in $\mathbb{R}$? Is A an F_σ-set? Justify your answer.

3. Define a metric 'd' on $\mathbb{N}$ such that no point of $(\mathbb{N}, d)$ is isolated. Justify your answer.

4. Let $A, B \subseteq \mathbb{R}$ where A is compact, B is closed and $A \cap B = \phi$. Prove that $d(A,B) > 0$.

5. Show that l_∞ is not separable.

Answer any two questions from Question Nos. 6 to 8 : [2×5]

6. Let $f_n(x) = \dfrac{1}{nx+1}$, $x \in (0,1)$, $n \in \mathbb{N}$

$g_n(x) = \dfrac{x}{nx+1}$, $x \in (0,1)$, check the pointwise and uniform convergence of f_n and g_n's in (0,1)

7. Let I be a bounded interval in $\mathbb{R}$ and let $\{f_n\}$, $\{g_n\}$ be two sequences of real valued functions, uniformly convergent on I. Let $h_n(x) = f_n(x) g_n(x)$, $x \in I$. Prove that, if for each $n \in \mathbb{N}, f_n, g_n$ are bounded in I, then h_n is uniformly convergent on I.

8. State and prove Dini's theorem for a sequence of real valued functions.

Group – B [25 marks]

Answer any three questions from Question Nos. 9 to 13 : [3×5]

9. Solve $x\dfrac{d^2y}{dx^2} - (x+2)\dfrac{dy}{dx} + 2y = x^3 e^x$ after the determination of a solution of its reduced equation.

10. Solve $(x+2)\dfrac{d^2y}{dx^2} - (2x+5)\dfrac{dy}{dx} + 2y = (x+1)e^x$ by the method of operational factors.

11. Find the eigen values and eigen functions of $\frac{d^2y}{dx^2}+\lambda y=0$ $(\lambda \neq 0)$ with y(0) = 0 and $y'(\ell)=0$.

12. Solve $\frac{dx}{y^3x-2x^4}=\frac{dy}{2y^4-x^3y}=\frac{dz}{9z(x^3-y^3)}$.

13. Solve $yz(y+z)dx+zx(x+z)dy+xy(x+y)dz=0$.

Answer <u>any two</u> questions from <u>Question Nos. 14 to 16</u> : [2×5]

14. Tangents are drawn from the origin to the curve y = sin x. Show that their points of contact lie on the curve $x^2y^2=x^2-y^2$.

15. If the polar equation of a curve is $r=f(\theta)$, where $f(\theta)$ is an even function of θ, show that the curvature at the point $\theta=0$ is $\frac{f(0)-f''(0)}{\{f(0)\}^2}$.

16. Show that the points common to the curve $2y^3-2x^2y-4xy^2+4x^3-14xy+6y^2+4x^2+6y+1=0$ and its asymptotes lie on the straight line $8x+2y+1=0$.

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